SOAL PERSIAPAN UAS KALKULUS II ITB

Soal :

1. Tentukan himpunan kekonvergensian untuk deret pangkat $\sum_{n=0}^{\infty} \frac{1}{n^3+1}$
2. Tentukan persamaan bidang yang mengandung garis x = 3t, y = 1 + t, z = 2t dan sejajar dengan perpotongan dari bidang-bidang 2x – y + z dan y + z + 1 = 0.
3. Sebuah kotak persegi panjang yang mempunyai tepi-tepi sejajar dengan sumbu-sumbu koordinatnya dimasukkan kedalam ellipsoid $36x^2 + 4y^2 + 9z^2 = 0$. Berapakah volume terbesar yang mungkin untuk kotak seperti ini.
4. Tentukan benda padat di oktan pertama yang dibatasi oleh silinder $y = x^2$ dan bidang x = 0, z = 0, dan y + z = 1.
5. Selesaikan:
   a. $x^2y'' + 5xy' + 4y = 0$ dengan terlebih dahulu mensubstitusikan $x = e^z$.
   b. $y'' - 2y' + y = x^2 + x$.

Jawaban :

1.

$$\rho = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^3+1} \div \frac{x^n}{n^3+1} \right|$$

$$= \lim_{n\to\infty} |x| \left| \frac{n^3+1}{(n+1)^3+1} \right| = |x|$$

When $x = 1$, the series is

$$\sum_{n=0}^{\infty} \frac{1}{n^3+1} = 1 + \sum_{n=1}^{\infty} \frac{1}{n^3+1} \le 1 + \sum_{n=1}^{\infty} \frac{1}{n^3}$$, which converges.

When $x = -1$, the series is $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3+1}$ which converges absolutely since $\sum_{n=0}^{\infty} \frac{1}{n^3+1}$ converges.

The series converges on $-1 \le x \le 1$.

2.

Using $t = 0$, one point of the plane is (0, 1, 0).
$\langle 2, -1, 1\rangle \times \langle 0, 1, 1\rangle = \langle -2, -2, 2\rangle = -2\langle 1, 1, -1\rangle$ is perpendicular to the normals of both planes, hence parallel to their line of intersection.
$\langle 3, 1, 2\rangle$ is parallel to the line in the plane we seek, thus $\langle 3, 1, 2\rangle \times \langle 1, 1, -1\rangle = \langle -3, 5, 2\rangle$ is a normal to the plane. An equation of the plane is
$-3(x-0) + 5(y-1) + 2(z-0) = 0$ or
$-3x + 5y + 2z = 5$.

3.

Let (x, y, z) denote the coordinates of the 1st octant vertex of the box. Maximize $f(x, y, z) = xyz$ subject to

$g(x, y, z) = 36x^2 + 4y^2 + 9z^2 - 36 = 0$

(where $x, y, z > 0$ and the box's volume is $V(x, y, z) = f(x, y, z)$.

Let $\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$.

$\langle yz, xz, xy \rangle 8 = \lambda \langle 72x, 8y, 18z \rangle$

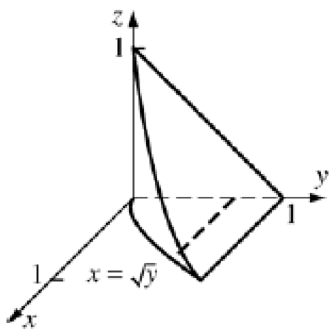
1. $8yz = 72\lambda x$
2. $8xz = 8\lambda y$
3. $8xy = 18\lambda z$
4. $36x^2 + 4y^2 + 9z^2 = 36$
5. $\frac{yz}{xz} = \frac{72\lambda x}{8\lambda y}$, so $y^2 = 9x^2$. (1, 2)
6. $\frac{yz}{xz} = \frac{72\lambda x}{18\lambda y}$, so $z^2 = 4x^2$. (1, 3)
7. $36x^2 + 36x^2 + 36x^2 = 36$, so $x = \frac{1}{\sqrt{3}}$. (5, 6, 4)
8. $y = \frac{3}{\sqrt{3}}$, $z = \frac{2}{\sqrt{3}}$ (7, 5, 6)

$V\left(\frac{1}{\sqrt{3}}, \frac{3}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right) = 8\left(\frac{1}{\sqrt{3}}\right)\left(\frac{3}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{16}{\sqrt{3}} \approx 9.2376$

The nature of the problem indicates that the critical point yields a maximum value rather than a minimum value.

4.



$$\int_0^1 \int_0^{\sqrt{y}} (1 - y)\,dx\,dy = \frac{4}{15}$$

5. A.

$$x^2 y'' + 5xy' + 4y = 0$$

Let $x = e^z$. Then $z = \ln x$;

$$y' = \frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} = \frac{dy}{dz}\frac{1}{x};$$

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}\left(\frac{dy}{dz}\frac{1}{x}\right) = \frac{dy}{dz}\frac{-1}{x^2} + \frac{1}{x}\frac{d^2y}{dz^2}\frac{dz}{dx}$$

$$= \frac{dy}{dz}\frac{-1}{x^2} + \frac{1}{x}\frac{d^2y}{dz^2}\frac{1}{x}$$

$$\left(-\frac{dy}{dz} + \frac{d^2y}{dz^2}\right) + \left(5\frac{dy}{dz}\right) + 4y = 0$$

(Substituting y' and y'' into (*))

$$\frac{d^2y}{dz^2} + 4\frac{dy}{dz} + 4y = 0$$

Auxiliary equation: $r^2 + 4r + 4 = 0$, $(r+2)^2 = 0$

has roots $-2, -2$.

General solution: $y = (C_1 + C_2 z)e^{-2z}$,

$$y = (C_1 + C_2 \ln x)e^{-2\ln x}$$

$$y = (C_1 + C_2 \ln x)x^{-2}$$

B.

Auxiliary equation: $r^2 - 2r + 1 = 0$ has roots 1, 1.

$$y_h = (C_1 + C_2 x)e^x$$

Let $y_p = Ax^2 + Bx + C$; $y_p' = 2Ax + B$;

$y_p'' = 2A$.

Then $(2A) - 2(2Ax + B) + (Ax^2 + Bx + C) = x^2 + x$.

$$Ax^2 + (-4A + B)x + (2A - 2B + C) = x^2 + x$$

Thus, $A = 1$, $-4A + B = 1$, $2A - 2B + C = 0$, so
$A = 1$, $B = 5$, $C = 8$.

General solution: $y = x^2 + 5x + 8 + (C_1 + C_2 x)e^x$